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Stress, Shock and the Sustainability of Optimal Resource Utilization in a Stochastic Environment

CHARLES PERRINGS

IIED
INTERNATIONAL INSTITUTE FOR ENVIRONMENT AND DEVELOPMENT

IIED/UCL LONDON ENVIRONMENTAL ECONOMICS CENTRE

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Charles Perrings

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London Environmental Economics Centre
IIED and University College London
3 Endsleigh Street
London WC1H ODD

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Author's Contact Address

The author may be contacted through:

London Environmental Economics Centre
3 Endsleigh Street
London
WC1H 0DD
U.K.

Tel: 071 388 2117
Fax: 071 388 2826
Tlx: 261681 EASCAN G

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CHARLES PERRINGS[1]

University of Auckland
and
London Environmental Economics Centre

Abstract

The degradation of resources in agriculture in many low income countries is increasingly argued to reflect an adverse economic environment - the product of ill advised intervention. Implicitly, this questions the compatibility of the economic and ecological environments within which economic activities take place. While this is an issue of general importance, it has attracted little attention in the past. This paper considers the theoretical problem of the role of an economic environment on optimal resource utilization in a stochastic ecological system. It does so in the context of a bioeconomic feedback control model of range management in semi-arid areas: a management problem characterized by a high level of uncertainty associated with high variance in rainfall, and the interdependence of the dynamics of herd size and carrying capacity. The paper pays particular attention to the conditions in which optimal resource utilization may be sustainable, and to the link between the unsustainable optimal policies and the economic environment.

Department of Economics
University of Auckland
Private Bag
Auckland
New Zealand
Telephone: 64.9.37 999
Fax: 64.9.366 0891

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[1] Thanks are due to Ed Barbier, Scott Barrett and Basil Sharp for helpful comments on an earlier draft of this paper.

1. Introduction

The close links between agricultural and economic growth in the low income countries of sub-Saharan Africa during a decade of 'crisis' has prompted a widespread theoretical and empirical reappraisal of the dynamics of agricultural sector performance in such countries. A common thread running through this reappraisal is the observation that falling agricultural productivity reflects the widespread degradation of the resource base. As Pearce et al [1988] put it: 'Africa's economic crisis certainly appears to be largely due to Africa's agricultural crisis', and 'Africa's agricultural crisis is in significant part an environmental crisis'. A variety of explanations for the trends that underlie this 'crisis' are offered in the literature. The dominant view, however, is that the causes of an ecologically unsustainable use of agricultural resources are to be found in the economic environment within which farmers make their decisions [cf Repetto, 1986,1989; Warford, 1989; Hodge, 1988; Lutz and El Serafy, 1989; Barbier, 1989].

However, while we may agree that the creation of an 'appropriate economic environment' is a precondition for the sustainable use of natural resources in agriculture, it is not at all clear what the term implies under the ecological conditions prevailing in sub-Saharan Africa - or anywhere else for that matter. There is a very strong tendency to equate 'an appropriate economic environment' with the liberalization of agricultural product and financial markets. Indeed, liberalization of agricultural prices in the low income countries has been widely endorsed on the grounds that it implies higher producer prices, and it is held that these will provide both the means and the incentive to conserve the agricultural resource base [cf Bond, 1983; Cleaver, 1985, 1988; Barbier, 1988, 1989]. But this endorsement is not founded on any clearly understood relation between the economic and ecological components of agricultural systems in the region. Whether any bioeconomic system is sustainable depends on both the ecological and the economic parameters of that system, and there is no reason to believe, a priori, that world prices are more compatible with the sustainability of the system than any other prices. The case for the liberalization of agricultural output prices in the low income countries may be strong, but it does not rest on an environmental foundation.

At the most general level, this paper is concerned with the constraints imposed by the economic environment on the sustainable use of ecological resources. To focus discussion, I consider the case of pastoralism in semi-arid areas such as the Sahel and much of eastern and southern Africa. The principal management problem in this case is to determine the optimal level of activity over time, given the evolution of the highly uncertain natural and economic environments within which pastoralists operate. Pastoralism may be said to be sustainable if the solution to this management problem does not involve the degradation of the natural resources required by the activity. Sustainability refers to the resilience of an ecological system subject to stress (determined by the level of economic activity) in the face of shocks (determined by climatic variation). This interpretation of sustainability, due to Conway and Barbier [Conway, 1986; Conway and Barbier, 1988; and Barbier, 1989a], directs our attention to the relation between the economic environment as the prime determinant of activity levels, and a stochastic ecological environment as the source of shocks.

The value of deterministic bioeconomic control models in the management of ecological resources in agriculture in the low income countries has recently been stressed by Barrett [1989a, 1989b]. Since an essential feature of rain-fed agriculture in semi-arid areas is that it is subject to a high degree of uncertainty related to the variance in rainfall, however, the management problem in this case is treated stochastically. More particularly, it is treated as an infinite horizon stochastic control problem, in which the state variables are the size of the herd and the carrying capacity of the range, and the control variable is the level of offtake. A stochastic control formulation turns out to be especially useful in analysing the long term effects of a given price structure under conditions of ecological uncertainty. While there is reason to believe that it is the long term effects of current price regimes that gives cause for alarm in the semi-arid lands, there are few attempts to incorporate those long term effects in an analysis of price policy. If it correctly describes the processes involved, the stochastic control model can identify the effect of a given price structure on the sustainability of the time paths of the state variables.

The paper is organised in seven sections. The following section discusses the ecological component of the model, and clarifies the assumptions made about range and herd dynamics in the absence of control. Section three elaborates the management problem addressed, and derives the equations required for the construction of an optimal policy. In section four a simulation is offered that permits description of the properties of an optimal policy in the stochastic case, and the properties of the time paths of both state and control variables under given values for the ecological and economic parameters. Sections five and six consider the impact of the economic environment within which the problem is optimized, and a final section offers some concluding remarks.

2. An ecological model

Three characteristics of the pastoral economy are assumed to be essential to the specification of the ecological component of a bioeconomic model of rangeland use:

1. Current changes in herd size and the level of offtake from herds affect future rangeland carrying capacity. This means both that the current carrying capacity of rangeland is not independent of the history of rangeland use, and that current herd size is not independent of the past evolution of carrying capacity. It is this interdependence of population and carrying capacity which makes the problem somewhat different from the many other renewable resource problems analysed in control terms.
2. The system evolves through periodic change. The 'process noise' that randomizes the time paths of herd size and carrying capacity is the product of variance in rainfall. Moreover, the effect of variance in rainfall on herd size and carrying capacity is not instantaneous, but occurs with a seasonally determined lag. Accordingly, time is treated discretely, rather than continuously, with a year being the natural interval. This assumption has certain fairly obvious implications for the time-behaviour of the system, given that difference and differential equations do not behave in the same way.
3. There is a very large measure of uncertainty attached to the future value of the state variables - herd size and carrying capacity. As has already been remarked, the carrying capacity of rangeland and the growth rate of the herd are both a function of rainfall, which has an extremely high variance in semi-arid areas. This characteristic makes the problem an intrinsically stochastic one.

To reflect these characteristics, the structure of the model developed here differs in certain respects from existing pastoral models. Like Barrett's [1989a] adaptation of the biological predator-prey models of May [1974], it postulates logistic growth functions for the herd and the vegetative cover of the range (which determines its carrying capacity), but as will become clear, treatment of the dynamics of carrying capacity is somewhat different.[1] Carrying capacity is assumed to change over time with changes

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[1] Barrett's [1989a] model has the following structure. The growth equation for the herd is

$$H_t = rH_t(1 - H_t/K_t) - h_t \qquad (1)$$

in which H_t denotes herd size at time t, r denotes the rate of growth of the herd, K_t denotes the carrying capacity of the range, and h_t denotes offtake at time t. The equation of motion for K_t is given by

$$K_t = a(\underline{K} - K_t) - bH_t \qquad (2)$$

in which $\underline{K}$ is defined as the 'saturation level of the grazing lands', a denotes the natural rate of regeneration of the rangeland, and b denotes the rate of depletion by the herd.

both in the degree of grazing pressure, and in climatic conditions. The relation between carrying capacity and grazing pressure is defined within the model. The relation between carrying capacity and factors exogenous to the model, such as rainfall, is captured by random variation of the ecological parameters of the model. Just how the regeneration of the range is expected to change with variation in endogenous factors will be discussed momentarily, but the important point is that the ecological model should reflect the fact that herd growth, the impact of herd size on rangeland vegetation, and the recuperative powers of the range are interdependent, and are all sensitive to climatic conditions.

The ecological system in isolation is assumed to be globally stable, in that the vegetative cover of the range is assumed to converge to some well-defined maximum value (the climax vegation) regardless of the severity of shocks due to either temporary climatic variation or overgrazing. This reflects the 'resilience' hypothesis [cf Walker and Noy-Meir, 1982], which rules out the possibility that there is some threshold level in the quality of the resource below which it will collapse completely. If the ecological system is globally stable, then it can withstand any level of overgrazing, regenerating in the same way irrespective of the damage inflicted on both the soil and its vegetative cover. Put another way, the assumption implies that nothing is irreversible in the ecological system: it is non-evolutionary. If variation in vegetative cover in the short run does not affect the climax vegetation in the long run this assumption is reasonable, but note that even in the short run it is not undisputed [cf Westoby, 1979]. It is worth emphasising that the assumption that the ecological system is globally stable does not imply that the pastoral economy is globally stable. Indeed, the collapse of carrying capacity and the extinction of the herd are possible outcomes of an optimal strategy.

Leaving the evolutionary nature of the ecological system to one side, and abstracting from the significance of herd and range composition, the remaining characteristics of pastoral systems seem to be adequately captured in the following equations of motion. The sequences $\{x_t\}$ and $\{k_t\}$ describe the time paths of the herd and the carrying

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It is assumed that the unit price of the offtake, p (> 0), and the unit cost of maintaining the herd, c (> 0), are both constant, implying that profits are given by $ph_t - cH_t$ for all t.. The economic control problem is then to

$$\max_{(h_t)} \int_0^{\infty} [ph_t - cH_t]e^{-\delta t}dt \qquad (3)$$

s.t. (1) and (2), and with $H_0, K_0 > 0$; $H_t, K_t \geq 0$; and $0 \leq h_t \leq h^{max}$. The last restriction implies that there is some maximum level of offtake which is different from the size of the herd itself, and which is constant over time. (3) is then solved for the optimal steady state offtake, and the optimal grazing pressure (H_t/K_t).

capacity of the range. Both are measured in terms of livestock units, and both may be thought of as the natural endogenous variables of the system. $\{x_t\}$ and $\{k_t\}$ are generated by the following first order forward recursions:

$$x_{t+1} - x_t = \alpha_t x_t(1 - x_t/k_t) - u_t \tag{1}$$
$$k_{t+1} - k_t = \beta_t k_t(1 - k_t/k_c) - \gamma_t(x_t - u_t) \tag{2}$$

in which

x_t = herd size at time t ($0 \leq x_t \leq k_c$);
k_t = carrying capacity at time t ($0 \leq k_t \leq k_c$);
k_c = maximum carrying capacity of the range;
u_t = offtake at time t ($-k_c \leq u_t \leq x_t$);
α_t = the net growth rate of the herd on the range whose use is being evaluated ($-1 \leq a_t$);
β_t = the rate of regeneration of the range ($-1 \leq b_t$);
γ_t = the rate of depletion of the range due to the herd ($\gamma_t \leq 1$).

Equation (1) describes the net growth of the herd in any given period as the difference between offtake and the natural growth of the herd, given the degree of grazing pressure, x_t/k_t. Equation (2) describes the net growth in the carrying capacity of the range as the difference between the net depletion of the vegetative cover of the range due to herd management policy, and the natural rate of regeneration of the range. Offtake, u_t, accordingly has both direct and indirect effects on the size of the herd. If livestock are drawn off in the current period, the size of the herd is reduced. But, at the same time, the future growth potential of the herd is improved due to the reduction in the net depletion of the vegetative cover of the range. Much of the interesting dynamics of the model are due to these indirect effects.

To obtain the maximum sustainable yield of the range given these equations of motion, notice first that the maximum carrying capacity of the range, k_c, is defined as the current carrying capacity of the climax vegetation on virgin range. At that point the growth in carrying capacity or the rate of regeneration of the range is zero, and the addition of any livestock will result in a fall in carrying capacity.[1] Maximization of the growth function (2) with respect to current carrying capacity shows that the maximum rate of

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[1] This abstracts from the potential for increasing carrying capacity through bush clearance - argued to be important in some areas.

regeneration of the range occurs when current carrying capacity is one half of the maximum carrying capacity: that is, when $k_t = k_m = 1/2k_c$. The maximum sustainable yield of the range is the point at which the net rate of depletion of the range is equal to the maximum rate of its regeneration. From (1) and (2), the size of the herd corresponding to the maximum sustainable yield is given by:

$$x_m = (k_c/4\alpha_t)\{-(1 - \alpha_t) \pm [(1 - \alpha_t)^2 + 2\alpha_t\beta_t/\gamma_t]^{1/2}\} \tag{3}$$

and the maximum sustainable level of offtake is given by

$$u_m = \alpha_t x_m(1 - x_m/k_m) = x_m - 1/2(\beta_t/\alpha_t)k_m \tag{4}$$

Whereas the state variables, x_t and k_t are restricted to non-negative values, offtake, u_t, may in principle be positive or negative. If offtake is positive (implying that livestock is being drawn off the range) it is limited to values less than or equal to the size of the herd. If offtake is negative (implying that the range is being restocked) it is limited to values less than or equal to the maximum carrying capacity of the range. Although a policy requiring negative offtake in the long run would not be economically interesting, restocking may be a part of an optimal strategy in the stochastic case.

The time behaviour of the ecological system depends on the ecological parameters, α_t β_t and γ_t. Note that the time behaviour of non-linear difference equations similar to (1) and (2) tends to be rather complex. Ignoring offtake, if a_t and b_t were assumed to be positive constant parameters (as they would be in the deterministic case), the size of the herd and the carrying capacity of the range would converge to equilibrium values so long as $0 < \alpha_t \beta_t \leq 2$. Moreover, convergence would be asymptotic for $0 < \alpha_t \beta_t \leq 1$, and through damped oscillation for $1 < \alpha_t \beta_t \leq 2$. But for $\alpha_t \beta_t \geq 2$ the sequences $\{x_t\}$ and $\{k_t\}$ would be non-convergent, and if $\alpha_t \beta_t > 2.57$, would exhibit 'chaotic' behaviour. In the general case where α_t and β_t are not restricted to positive values, and are time-varying, the properties of the system will change depending on the current value of α_t. $\{x_t\}$ and $\{k_t\}$ may converge on some positive growth path over some time segments, may converge on zero over others, or may be entirely non-convergent.

In this paper, α_t β_t and γ_t are defined as stochastic parameters. That is, they are independently distributed random numbers with means $\underline{\alpha}$, $\underline{\beta}$ and $\underline{\gamma}$, and variances σ_α^2,

σ_β^2 and σ_γ^2. The system is thus subject to 'process noise'.[1] To get a sense of the likely values for the mean and variance of α_t, we need to consider both the ecological and institutional determinants of herd growth. Recall that the focus of the paper is rangeland in semi-arid areas in the low income countries of sub-Saharan Africa. Institutionally, these areas tend to be dominated by more or less regulated common property regimes. While the main source of variance in α_t is rainfall (which affects the net natural rate of increase of the herd), it also depends on the incidence of disease (which need bear no particular relation to the level of rainfall).[2] The net natural rate of increase is a function both of fertility and mortality in the herd, each of which tends to be sensitive to the level of rainfall for most species herded in these areas. Pastoralism in most semi-arid areas involves the managed movement of herds between ranges, depending on the state of the vegetative cover - the pattern being facilitated by open access common property regimes. But there is also some autonomous movement both of livestock and of competing ungulates. This implies that while $\underline{\alpha}$ will tend to have a value somewhere near the long-run mean net natural rate of increase of the herd itself, it will be subject to considerable variance. Indeed, given the influence that rainfall has on herd fertility, mortality and migration, fluctuation in rainfall has historically led to dramatic swings in herd sizes on a given range from one period to the next. It has also led to dramatic shifts in the species composition of the herd, although no attempt is being made to model this here.

The mean and variance of β_t capture the net natural rate of increase of the vegetation consumed by the herd on the rangeland of interest. As in the case of herd growth, there are a number of different effects involved here. Clearly, there is a positive correlation between rainfall and the growth of graze or browse, but where there is a short run shift in species composition within the vegetative cover from edible grasses to woody biomass, or from edible to non-edible grasses, this will show up as a decrease in vegetative cover, even though total biomass may have increased. Similarly, where climatic conditions associated with increasing vegetative biomass also favour the growth of populations of competitors to the herd (other ungulates or insects, say), the graze or browse available to the herd may decrease. This is a very real problem in many of the semi-arid areas of sub-saharan Africa, which are also populated by highly mobile, highly fluctuating herds of antelope, and are subject to depredation by insect swarms. An additional complication arises if edible grasses are not the climax vegation of the area, but this problem is set aside in this paper. Once again, while $\underline{\beta}$ will tend to

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[1] In reality grazing systems are usually also subject to 'observation noise'. That is, the state variables x_t and k_t will tend to be observed only with some error. However, we may ignore this additional source of uncertainty here.

have a value somewhere near the long-run, mean, net natural rate of regeneration of the range, it will be subject to considerable variance.

As a result of the variance of α_t and β_t, the time behaviour of the recursions (1) and (2) may be extremely complex - even in the absence of offtake. The herd growth function may have normal compensatory, overcompensatory, depensatory and critical depensatory properties for similar herd sizes at different periods. There is no reason to believe that normal compensatory growth (which leads asymptotically to convergence to equilibrium values for both herd size and range carrying capacity) will be encountered in reality. Indeed, it is more likely that growth will be overcompensatory (leading either to convergence via damped oscilliations, or to non-convergent oscillation). But it is also quite possible for the growth function to be critically depensatory (leading to the collapse of the herd) where carrying capacity falls sharply over consecutive periods. In general, change in the size of the herd will vary directly with change in the level of grazing pressure given by the ratio x_t/k_t. In general, that is, overgrazing will lead to a decline in the size of the herd. However, it is important to add that since the natural rate of growth of both herd and carrying capacity is assumed to fluctuate, and since negative values for α_t and β_t are admissible, this will not necessarily be the case.

There are two broad senses in which the range may be overgrazed: economically and ecologically. Economic overgrazing is discussed in section 3 below. Ecological overgrazing can also be defined in two rather different ways. Ecological overgrazing may be said to be either 'fundamental' or 'current'. Fundamental ecological overgrazing may be said to exist whenever the stochastic equilibrium level of grazing pressure exceeds the level of grazing pressure corresponding to the maximum sustainable yield of the range: that is, when $\lim_{t\to\infty} x_t/k_t > x_m/k_m$. This corresponds to the meaning given to ecological overgrazing by, for example, Barrett [989a]. Current ecological overgrazing may be said to exist whenever the current level of grazing pressure exceeds the maximum sustainable grazing pressure: that is, when $x_t/k_t > x_m/k_m$. Given the method of solution adopted here, we shall tend to focus on the more limited current ecological overgrazing (hereafter referred to as 'ecological overgrazing').

In the absence of offtake/restocking, the difference $x_{t+1} - x_t$ will be negative if $a_t x_t(1 - x_t/k_t) < 0$, which will occur either if $(1 - x_t/k_t) < 0$ and $\alpha_t > 0$; or if $(1 - x_t/k_t) > 0$ and $\alpha_t < 0$. The first alternative implies that the future size of the herd will decline where pressure is increasing on range that is currently being overgrazed. The second option implies that future herd sizes may fall (due to disease or drought, say) where the range is not currently being overgrazed. Moreover, since $x_{t+1} - x_t > 0$ if $(1 - x_t/k_t) < 0$ and α_t

< 0, future herd sizes may rise where the range is being currently overgrazed in an ecological sense, providing that herd pressure on the range is falling. Similarly, the carrying capacity of the range in the uncontrolled case may decline for various reasons. $\beta_t k_t(1 - k_t/k_c) - \gamma_t x_t < 0$ if either $\beta_t > 0$ and depletion is greater than regeneration, or if $\beta_t < 0$. The fact that herd size may exceed the carrying capacity of the range in any one year is not, therefore, a necessary condition for declining carrying capacity in the next year. Current ecological overgrazing may not imply fundamental ecological overgrazing.

3 The management problem

The optimal size of the herd relative to the carrying capacity of the range - the optimal level of grazing pressure - is the solution to a problem of decision-making under incomplete information. To facilitate construction of this problem we may assume that only the ecological parameters of the system are not known with certainty. Output prices are initially assumed to be determined exogenously, and to be constant over time. This is, in fact, a reasonable assumption to make with respect to some agricultural products - beef exports on long term contracts under the Lomé convention, for example. In general, however, the volatility of agricultural prices is a key factor in the sustainability in agriculture, particularly in the wake of price liberalization, and we shall return to this problem later. Incomplete information in the model accordingly refers to ignorance about the time trends of the physical system. The optimal policy is one that maximizes the expected welfare deriving from pastoral activity over an infinite horizon through choice of the level of offtake, and subject to the properties of the physical system. Such a policy may be said to be sustainable if it preserves the options available to future resource users by ensuring that the resource base is not depleted over time in the face of climatic (and other) shocks. More formally, the problem is to:

$$\max_{\{u_t\}} E\left[\sum_{t=0}^{\infty} \rho^t W(x_t, k_t, u_t)\right] \qquad \text{(5a)}$$

s.t

$$x_{t+1} - x_t = \alpha_t x_t(1 - x_t/k_t) - u_t \qquad \text{(5b)}$$

$$k_{t+1} - k_t = \beta_t k_t(1 - k_t/k_c) - \gamma_t(x_t - u_t) \qquad \text{(5c)}$$

$$x_0\ (> 0\ ) = x(0) \qquad \text{(5c)}$$

$$k_0\ (> 0\ ) = k(0) \qquad \text{(5d)}$$

$$u_t \leq x_t \qquad \text{(5e)}$$

$$x_t, k_t \geq 0 \qquad \text{(5f)}$$

where E denotes expected value, and where $\rho = [1/(1+\delta)]$ denotes a discount factor, with δ being the rate of discount. There is no explicit sustainability constraint involved in this infinite horizon version of the problem.[1] However, the link between the optimal policy and the sustainability of the use of the resources involved will become clear.

The solution to this problem comprises a decision rule which fixes the optimal offtake policy for the given parameter values, and for the current values of the state variables. The rule is derived for the stochastic equilibrium of the system, and is applied sequentially to the actual values of the state variables, as these are observed, in a form of closed loop, or feedback, control. Derivation of the decision rule depends on the mean values of the stochastic parameters and accordingly abstracts from the uncertainty (process noise) generated by the random variation of the growth coefficients, α_t and β_t, and the rate of depletion, γ_t.

Without specializing the welfare function further at this stage, we may identify the necessary conditions for the control sequence $\{u_t\}$ to be optimal at the stochastic equilibrium of the system. Defining the current value Hamiltonian in terms of the expected values of $\underline{\alpha}$, $\underline{\beta}$ and $\underline{\gamma}$:

$$\begin{aligned} H(x_t,u_t,\lambda_t) = W(x_t,k_t,u_t) + \rho\lambda_{t+1}\{\underline{\alpha}x_t[1 - x_t/k_t) + k_t)] - u_t\} + \\ \rho\zeta_{t+1}\{\underline{\beta}k_t(1 - k_t/k_c) - \underline{\gamma}(x_t - u_t)\} \end{aligned} \tag{6a}$$

the first order conditions require that

$$\begin{aligned} 0 &= H_{ut} &&= W_{ut} - \rho\lambda_{t+1} + \rho\zeta_{t+1}\underline{\gamma}x_t/x_m && (6b)\\ \rho\lambda_{t+1} - \lambda_t &= -H_{xt} &&= -W_{xt} - \rho\lambda_{t+1}\underline{\alpha}(1 - 2x_t/k_t) + \rho\zeta_{t+1}\underline{\gamma} && (6c)\\ \rho\zeta_{t+1} - \zeta_t &= -H_{kt} &&= -W_{kt} - \rho\lambda_{t+1}\underline{\alpha}x_t^2/k_t^2 - \rho\zeta_{t+1}\underline{\beta}(1 - 2k_t/k_c) && (6d)\\ x_{t+1} - x_t &= H_{\rho\lambda t+1} &&= \underline{\alpha}x_t(1 - x_t/k_t) - u_t && (6e)\\ k_{t+1} - k_t &= H_{\rho\zeta t+1} &&= \underline{\beta}k_t(1 - k_t/k_c) - \underline{\gamma}(x_t - u_t) && (6f)\\ x_0 &= x(0) &&&& (6f)\\ k_0 &= k(0) &&&& (6g)\\ u_t &\in U &&&& \end{aligned}$$

The maximum condition (6b) requires that at the optimal level of offtake the marginal direct benefit of livestock sales, W_{ut}, should be equal to the discounted intertemporal

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[1] In a finite horizon problem an arbitrary terminal value of these variables may be assigned.

'shadow price' of offtake, $\rho\lambda_{t+1} + \rho\zeta_{t+1}\gamma$. The breakdown of this shadow price is quite intuitive. The first part of the expression, $\rho\lambda_{t+1}$, is just the cost of offtake in terms of the growth of the herd. It is the discounted value of future livestock units foregone by drawing down the herd in the current period.[1] The second part, $\rho\zeta_{t+1}\gamma$, is the gain of offtake in terms of future herd growth, attributable to the effects of lower herd densities on range regeneration. The full expression $\rho\lambda_{t+1} + \rho\zeta_{t+1}\gamma$ is therefore the discounted net user cost of offtake. The maximum condition accordingly requires that the marginal direct benefit of livestock sales should be equal to the discounted net user cost of offtake.

The adjoint equations, (6c) and (6d), describe the evolution of the shadow prices of the state variables as a function of offtake policy and the dynamics of the physical system. If a steady state solution (stochastic equilibrium) exists, such that $\lambda_t = \lambda_{t+1}$, and $\zeta_t = \zeta_{t+1}$, for $t = k, k+1, \ldots$, (6b), (6c) and (6d) may be used to define a steady state 'rule' for determining the optimal level of grazing pressure. To simplify, let us first define the ratios:

$$\kappa_t \equiv k_t/k_c \tag{7a}$$
$$\psi_t \equiv x_t/k_t \tag{7b}$$
$$\omega_{xt} \equiv W_{xt}/W_{ut} \tag{7c}$$
$$\omega_{kt} \equiv W_{kt}/W_{ut} \tag{7d}$$

The first two have already been discussed: (7a) denotes the ratio of current carrying capacity relative to the maximum carrying capacity; and (7b) is the index of grazing pressure. (7c) and (7d) give the ratios of the marginal costs of, respectively, livestock and carrying capacity, to the marginal benefit of offtake. Once we specialize the welfare function these ratios will have the natural interpretation of the real-product cost of livestock maintenance and carrying capacity.Solving for the steady state values of λ and ζ from (6c) and (6d), inserting these into (6b), and using (7) yields the quadratic equation:

$$0 = \psi_t{}^{*2}\,\underline{\alpha\gamma}(1+\omega_{xt}) - \psi_t{}^{*}2\,\underline{\alpha}[\underline{\beta}(1-2\kappa_t) - \omega_{kt}\gamma - \delta] + \omega_{kt}\gamma(1 - \underline{\alpha} + \delta) + [\underline{\beta}(1-2\kappa_t) - \delta][\omega_{xt} + \underline{\alpha} - \delta] \tag{8}$$

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[1] Since the herd is assumed to be homogeneous for purposes of this paper, offtake of any livestock units has the same effect on future herd growth. This abstracts from the specialized roles of livestock units by age and gender. Naturally, in the management of both domestic livestock and wildlife, the direct future costs of offtake in terms of herd growth can be minimized by selection of the units to be withdrawn from the range.

ψ_t^*, a positive root of (8), denotes the optimal level of grazing pressure.

It is now possible to be more precise about the concepts of overgrazing already discussed. Two measures of overgrazing can be identified: a measure of economic overgrazing, ξ_t, and a measure of ecological overgrazing, χ_t. These are defined as follows:

$$\xi_t = (\psi_t/\psi_t^*) - 1 \tag{9a}$$
$$\chi_t = (\psi_t/\psi_m) - 1 \tag{9b}$$

where ψ_m denotes the maximum sustainable grazing pressure - the grazing pressure at the maximum sustainable yield. Overgrazing in an economic sense may be said to exist only if the actual level of grazing pressure exceeds the optimal level of grazing pressure ($\xi_t > 0$), whereas overgrazing in an ecological sense may be said to exist only if the actual level of grazing pressure exceeds the maximum sustainable grazing pressure ($\chi_t > 0$). Notice that the optimal level of grazing pressure is not necessarily the same as the maximum sustainable grazing pressure. Whether optimal grazing pressure is greater or less than the maximum sustainable grazing pressure depends on the economic parameters of the system - the relative prices facing resource users. If relative prices are such that it is optimal to 'mine' the range, the optimal grazing pressure will exceed the maximum sustainable grazing pressure - implying that there will be fundamental ecological overgrazing. On the other hand if relative prices are consistent with the sustainable use of the resource, the optimal grazing pressure will be less than or equal to the maximum sustainable grazing pressure. We shall return to this point later.

The construction of a decision rule for the level of offtake from (8) requires one further step. The optimal grazing pressure in any given period is determined by reference to the mean value of the economic and ecological parameters of the system, and to the current carrying capacity of the range, k_t. Given ψ_t^* the optimal herd size corresponding to k_t is obtained directly as $\psi_t^* k_t$, and the expected steady state offtake corresponding to this herd size is simply $\alpha\psi_t^* k_t(1-\psi^*)$. For a system away from the steady state this requires some modification. In the general case it may be expressed as:

$$u_t{}^* = \alpha\psi_t^* k_t(1-\psi^*)(1 + \xi_t) \tag{10}$$

implying that offtake will be higher (or lower) than the steady state level in proportion to the degree of economic overgrazing (or understocking) in the system. Substitution of the optimal grazing pressure into (8) yields optimal offtake.

Equations (8) and (10) provide the basis for a sequential feedback control policy in which the level of offtake in any given period is determined for the observed values of the state variables, and for the expected values of the stochastic ecological parameters. If the bioeconomic system is dynamically stable, then grazing pressure in a system subject to such a control policy will tend towards the optimal level - the stochastic equilibrium of the system. As we shall see, however, whether the bioeconomic system is stable depends on the economic environment within which the resource is used. The characteristics of the control sequence under a range of economic conditions are illustrated in the following simulation.

4. A simulation

To illustrate the construction of an optimal policy, we need to specialize the welfare function further. It is typically assumed that welfare in the rural economy is a function of farm profit. This implies that it takes the following simple additive separable form:

$$W(u_t,x_t,k_t) = pu_t - cx_t - rk_t \tag{11}$$

in which p denotes the constant slaughter price of offtake, c denotes the constant cost of livestock maintenance, and r denotes the constant cost of carrying capacity. r may be thought of as a productivity-related charge for the use of grazing land, or grazing fee. Since the main concern of this paper is with the implications of a given economic environment for the optimal management of rangeland under ecological uncertainty, maximization of (11) is acceptable as a first approximation of the social objective function. There would, however, be very serious reservations about its use in the case of the pastoral economy in semi-arid low income countries.

First, the additive separable form of the function implies a neutral attitude towards risk that is inappropriate in these conditions. Second, it ignores transfers. In many cases of interest profit may be negative for most prices compatible with the sustainable use of rangeland - as is the case in many pastoral economies in sub-Saharan Africa where transfers account for a major part of rural incomes. Third, the assumption that the relative prices are constant over time is obviously too strong. Fourth, and most important, the role of livestock in pastoral economies in the low income countries goes far beyond the production of beef, sheepmeat or goatmeat. The maintenance of livestock does involve costs (which in reality are an increasing function of the level of grazing pressure), but it also provides significant benefits to its owners in the form of draft power, animal products, the status it confers, the insurance it provides against adverse climatic conditions, and the fact that it is 'privileged currency' in bridewealth and other important social transactions. What c approximates in (11), therefore, is the net unit maintenance cost of livestock. Given (11) it follows that the ratios ω_{xt} and ω_{kt} are just the marginal real-product cost of livestock maintenance, and the marginal real-product cost of carrying capacity.

This exercise is not intended to capture the behaviour of any given pastoral economy, even though the parameter values are based on data for the livestock sector in

Botswana[1]: a good example of a case where control takes place in the context of an ecological system driven by fluctuating levels of rainfall. In this exercise, the sequences $\{\alpha_t\}$, $\{\beta_t\}$ and $\{\gamma_t\}$ are random numbers with means:

$\alpha = 0.24$
$\beta_t = 0.03$
$\gamma = 0.06$

and coefficients of variation

$\sigma_\alpha/\alpha = 0.65593$
$\sigma_\beta/\beta = 0.25092$
$\sigma_\gamma/\gamma = 0.2075.$

The value for the relative variance of $\{\beta_t\}$ derives from rainfall data for the Maun district, while $\{\alpha_t\}$ and $\{\gamma_t\}$ have the same relative variance as the average rate of growth of livestock in that country. These sequences are recorded in table 1.

The initial conditions in the example have been selected to correspond to the case where a herd of unsustainable size is introduced to a range regenerating at the maximum rate. This implies that there is ecological overgrazing (in the sense that the current level of grazing pressure exceeds the level of grazing pressure associated with the maximum sustainable yield of the range). Initial values of x_t and k_t are:

$x_0 = 50$
$k_0 = 100$

The maximum carrying capacity of the range, k_m, is assumed to be 200. Given these values, the level of grazing pressure corresponding to the maximum sustainable yield of the range is $\psi_m = 0.335$, and the degree of (fundamental) ecological overgrazing is $\chi_t = 0.4992$. The implication of these initial conditions is that if the herd were not controlled (if there was no offtake) ecological overgrazing would lead to a 'Malthusian' collapse of the herd. This is illustrated in Figure 1, which shows the time paths for the state variables, x_t and k_t, in the absence of any control ($u_t = 0$ for all t). In this and all

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[1] See See Botswana, Central Statistics Office. [1988], 1985/6 Household Income and Expenditure Survey. Also Botswana, Central Statistics Office. [1988] and Botswana, Ministry of Agriculture. [1987]. Current Botswana series do not provide direct estimates of rates of rangeland depletion or regeneration, so the mean values of β and γ reflect traditional differences in the duration of periods of land use and fallow in the semi-arid zones of sub-Saharan Africa [Allan, 1965].

subsequent figures, time is registered on the horizontal axis, the values on that axis being equal to t+1.

Table 1: **Values for $\{\alpha_t\}$ $\{\beta_t\}$ and $\{\gamma_t\}$**

time (t + 1)	α_t	β_t	η_t
1	0.37806	0.01885	0.04889
2	0.58355	0.02990	0.04309
3	0.27315	0.02342	0.04293
4	-0.01060	0.03973	0.06782
5	0.12203	0.02916	0.06052
6	0.36677	0.03096	0.08122
7	0.24842	0.03780	0.06003
8	0.07811	0.03282	0.04219
9	0.24325	0.04946	0.06188
10	0.36018	0.01359	0.07539
11	0.17217	0.02911	0.04828
12	0.38300	0.02151	0.04501
13	0.20607	0.03641	0.05170
14	0.12335	0.02297	0.06526
15	0.33795	0.02666	0.06523
16	0.14020	0.02733	0.04943
17	0.18917	0.03488	0.07395
18	0.24764	0.02719	0.07185
19	0.02067	0.01972	0.05583
20	0.11453	0.02607	0.07373
21	0.36579	0.03719	0.06513
22	0.04776	0.03320	0.06025
23	0.11209	0.02710	0.05620
24	0.39499	0.04084	0.05740
25	0.58355	0.02990	0.04309
26	0.27315	0.02342	0.04293
27	-0.01060	0.03973	0.06782
28	0.12203	0.02916	0.06052
29	0.36677	0.03096	0.08122
30	0.36677	0.03096	0.08122

To address the economic problem, we need to add information on the non-stochastic economic parameters. These have also been constructed from Botswana data, although no claim is made for their accuracy. The slaughter price of offtake, p, is taken as the numeraire. The constant cost of maintaining cattle on the range in terms of the slaughter price is assumed to be 0.05, and the implicit cost of accessing the range in terms of p is assumed to be 0.03. The control policy in this case is constructed sequentially. Given the initial value of the state variables, x_0 and k_0, the optimal offtake, u_0^*, is calculated from (8) (9) and (10) on the basis of the expected values of the stochastic ecological parameters, $\underline{\alpha}$, $\underline{\beta}$ and $\underline{\gamma}$, and the known values of the economic parameters, p, c and r.

Using ex post observations on the actual values, α_0, β_0 and γ_0, and the equations of motion, (1) and (2), we may determine values for x_1 and k_1. These values are substituted into (8) to obtain ψ_1*, and into (10) to obtain u_1*, and so on.

Figure 1: Time paths for x_t and k_t (uncontrolled case).

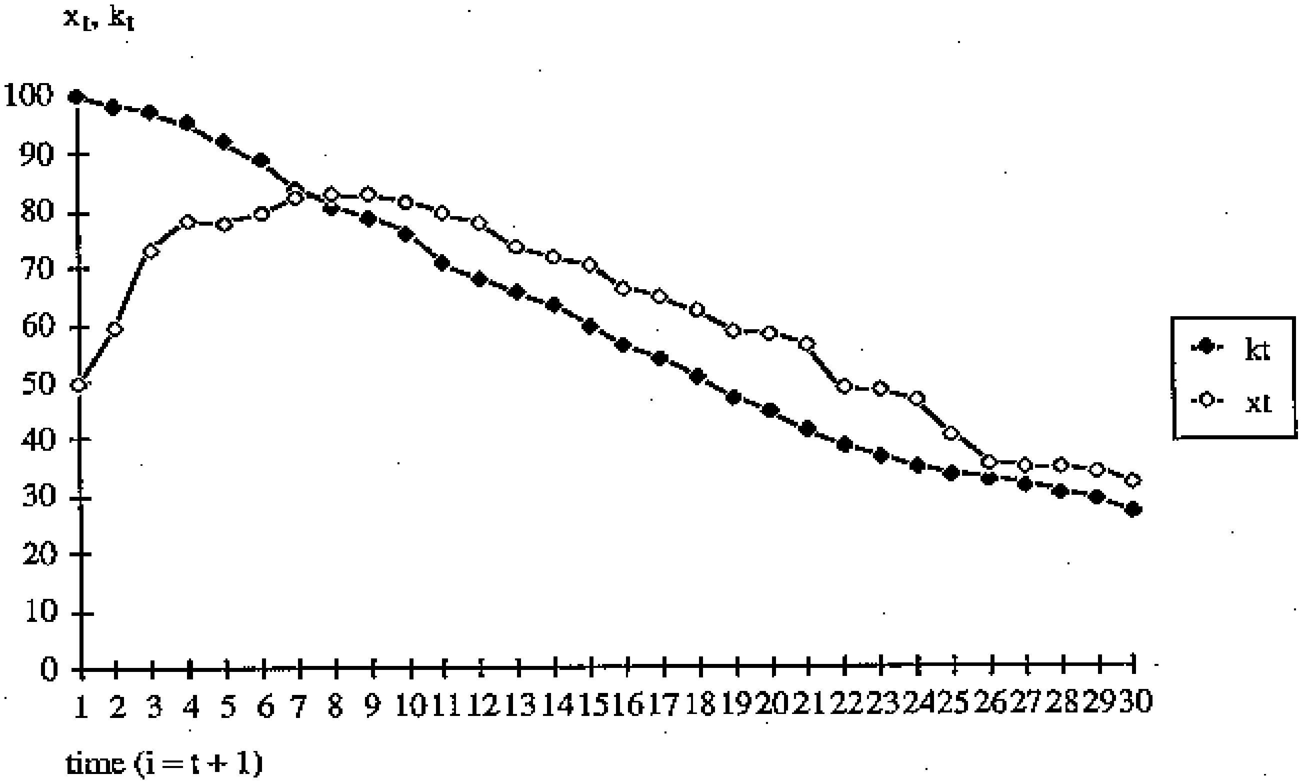

Table 2 reports the values for the state and control variables over the first thirty periods in the control sequence, together with (undiscounted) values for the maximand (farm profits), and the two measures of overgrazing in the system.

Table 2: Time paths for state and control variables

time	k_t	x_t	u_t	Y_t	x_t/k_t	ξ_t	χ_t
1	100.000	50.000	8.290	2.790	0.500	0.431	0.499
2	98.903	51.160	8.482	2.957	0.517	0.498	0.566
3	98.559	57.090	9.464	3.653	0.579	0.690	0.764
4	97.686	54.186	8.982	3.342	0.554	0.634	0.702
5	96.606	44.948	7.450	2.304	0.465	0.392	0.445
6	95.792	40.431	6.700	1.805	0.422	0.276	0.323
7	94.598	42.300	7.009	2.056	0.447	0.361	0.406
8	94.364	41.099	6.810	1.924	0.435	0.333	0.375
9	94.553	36.101	5.982	1.340	0.381	0.178	0.215
10	95.155	35.547	5.891	1.259	0.373	0.155	0.194

Table 2: Time paths for state and control variables (continued)

time	k_t	x_t	u_t	Y_t	x_t/k_t	ξ_t	χ_t
11	93.597	37.677	6.242	1.551	0.402	0.253	0.288
12	93.529	35.309	5.850	1.279	0.377	0.181	0.213
13	93.274	37.877	6.275	1.583	0.406	0.270	0.304
14	93.452	36.237	6.004	1.388	0.387	0.216	0.249
15	92.623	32.970	5.462	1.035	0.356	0.127	0.154
16	92.154	34.683	5.745	1.246	0.376	0.193	0.219
17	92.082	31.970	5.296	0.935	0.347	0.107	0.130
18	91.843	30.622	5.072	0.786	0.333	0.068	0.089
19	91.358	30.604	5.069	0.798	0.335	0.077	0.096
20	90.911	25.955	4.299	0.274	0.285	-0.072	-0.057
21	90.607	23.780	3.938	0.031	0.262	-0.140	-0.128
22	91.158	26.257	4.349	0.301	0.288	-0.060	-0.045
23	91.485	22.801	3.776	-0.107	0.249	-0.181	-0.168
24	91.761	20.942	3.469	-0.330	0.228	-0.246	-0.234
25	92.786	23.857	3.952	-0.023	0.257	-0.156	-0.139
26	93.416	30.247	5.011	0.696	0.323	0.053	0.076
27	93.499	30.822	5.106	0.760	0.329	0.071	0.094
28	93.733	25.497	4.224	0.137	0.272	-0.106	-0.087
29	93.897	23.537	3.900	-0.093	0.250	-0.172	-0.154
30	93.845	26.106	4.325	0.205	0.278	-0.082	-0.063

The first point to note about this table is that at the parameter values assumed here the activity is sustainable over the time horizon of interest. The optimal grazing pressure converges to a value that is less than the maximum sustainable grazing pressure. The average value of χ_t is negative for $t > 19$. In other words, it is not optimal to 'mine' the range. Moreover, the net present value of the income stream is positive at a discount rate of 5% over the whole time horizon. Nevertheless, the activity is clearly marginal with farm profits at or near the stochastic equilibrium oscillating around zero.

The time paths for the state and control variables are shown in figure 2. Carrying capacity and herd size are registered on the left hand vertical axis, and offtake is registered on the right hand vertical axis. The time paths for the state variables in this case are asymptotically convergent on their stochastic equilibrium values, the fluctuation in herd size being attributable to variation in the parameter α_t. Indeed, a control policy that is sensitive to change in current growth rates for the herd can significantly smooth the path of $\{x_t\}$.[1] Even with fluctuating herd sizes, however, this offtake policy effectively protects against degradation of the range.

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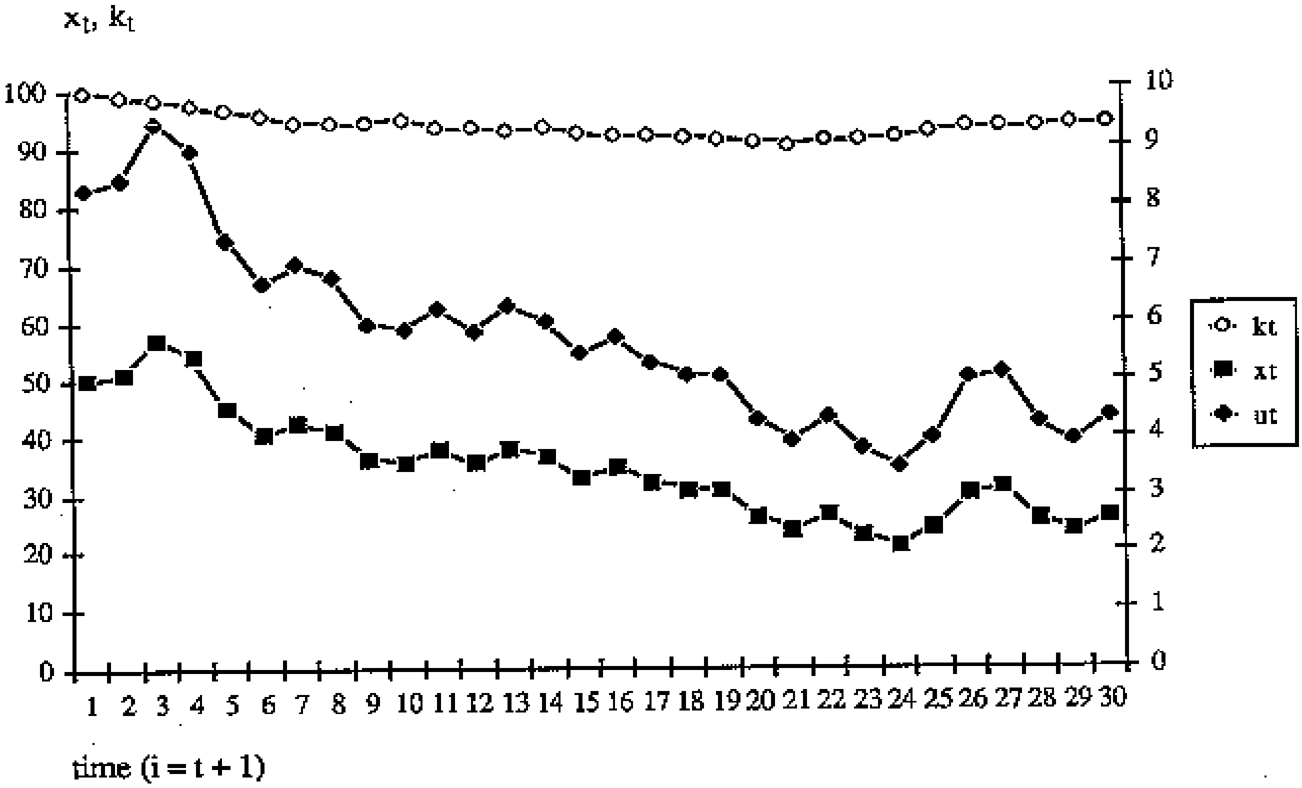
[1] Such a policy requires that $u_t{}^* = \alpha_t\psi_t{}^*k_t(1-\psi^*)(1 + \xi_t)$, with α_t replacing $\underline{\alpha}$.

Figure 2: Time paths for x_t and k_t (controlled case).



5. The economic environment and ecological stress

The question that motivates the paper is how the sustainability of resource utilization is related to the economic environment. More particularly, we are interested in the ecological impacts of a change in agricultural prices and income. This section first considers the relationship between the state variables and the system parameters of the general model, and then extends the simulation of section four to indicate the impact of specific change in the key economic parameters.

Recall that the sustainability of the ecological system underpinning pastoral activity implies its resilience to shock under the stress imposed by the level of that activity, where shock is the product of climatic perturbation and stress is indicated by the level of grazing pressure. In general, shocks may be due to either climatic or economic perturbation, but the latter case is not considered here.[1] The resilience of the ecological system is accordingly indicated by the response of the state variables (under an optimal

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[1] Since the relation between economic and ecological parameters is perfectly symmetrical, nothing is lost by this restriction.

policy) to perturbation of the ecological parameters. Non-resilience implies the collapse of the carrying capacity of rangeland, or the herd, or both.

The first point to make here is that the response of the system to exogenous shock does not depend on current levels of grazing pressure alone. From (8), (1) and (2), the change in the optimal level of grazing pressure in this period due to change in the ecological parameters in the last period is given by:

$$\frac{d\psi_t^*}{d\alpha_{t-1}} = -\frac{2\underline{\alpha}[\psi_t^*\underline{\gamma}(1+\omega_{xt}) - [\underline{\beta}(1-2\kappa_t) - \omega_{kt}\underline{\gamma} - \delta][x_{t-1}(1-\psi_{t-1})]}{k_t V}$$

$$\frac{d\psi_t^*}{d\beta_{t-1}} = -\frac{[\psi_t^{*2}\underline{\alpha}\underline{\gamma}(1+\omega_{xt}) - 2\psi_t^*\underline{\alpha}k_t(\underline{\beta}(1-2\kappa_t) - \omega_{kt} - \delta) + 2\underline{\beta}(\underline{\alpha}(2\psi_t^*-1) - \omega_{kt} - \delta)(k_t^2/k_c)][k_{t-1}(1-k_{t-1})]}{k_t^2 V}$$

$$\frac{d\psi_t^*}{d\gamma_{t-1}} = -\frac{[\psi_t^{*2}\underline{\alpha}\underline{\gamma}(1+\omega_{xt}) - 2\psi_t^*\underline{\alpha}k_t(\underline{\beta}(1-2\kappa_t) - \omega_{kt} - \delta) + 2\underline{\beta}(\underline{\alpha}((2\psi_t^*-1) - \omega_{kt} - \delta)(k_t^2/k_c)][\gamma_{t-1}(x_{t-1}-u_{t-1}))]}{k_t^2 V}$$

where

$$V = 2\underline{\alpha}[\psi_t^{*2}\underline{\gamma}(1+\omega_{xt}) - \psi_t^* k_t(\underline{\beta}(1-2\kappa_t) - \omega_{kt} - \delta) \tag{12}$$

How the optimal level of grazing pressure changes as a result of perturbation of the ecological parameters at time t-1 depends, as one would expect, on the value of the state and control variables, k_{t-1}, x_{t-1} and u_{t-1} (and the mean value of the ecological parameters, $\underline{\alpha}$, $\underline{\beta}$ and $\underline{\gamma}$). But it also depends on the value of the economic parameters at time t, ω_{xt} and ω_{kt}: (the ratio of current input to output prices under the welfare function assumed here), and δ (the current discount or interest rate).

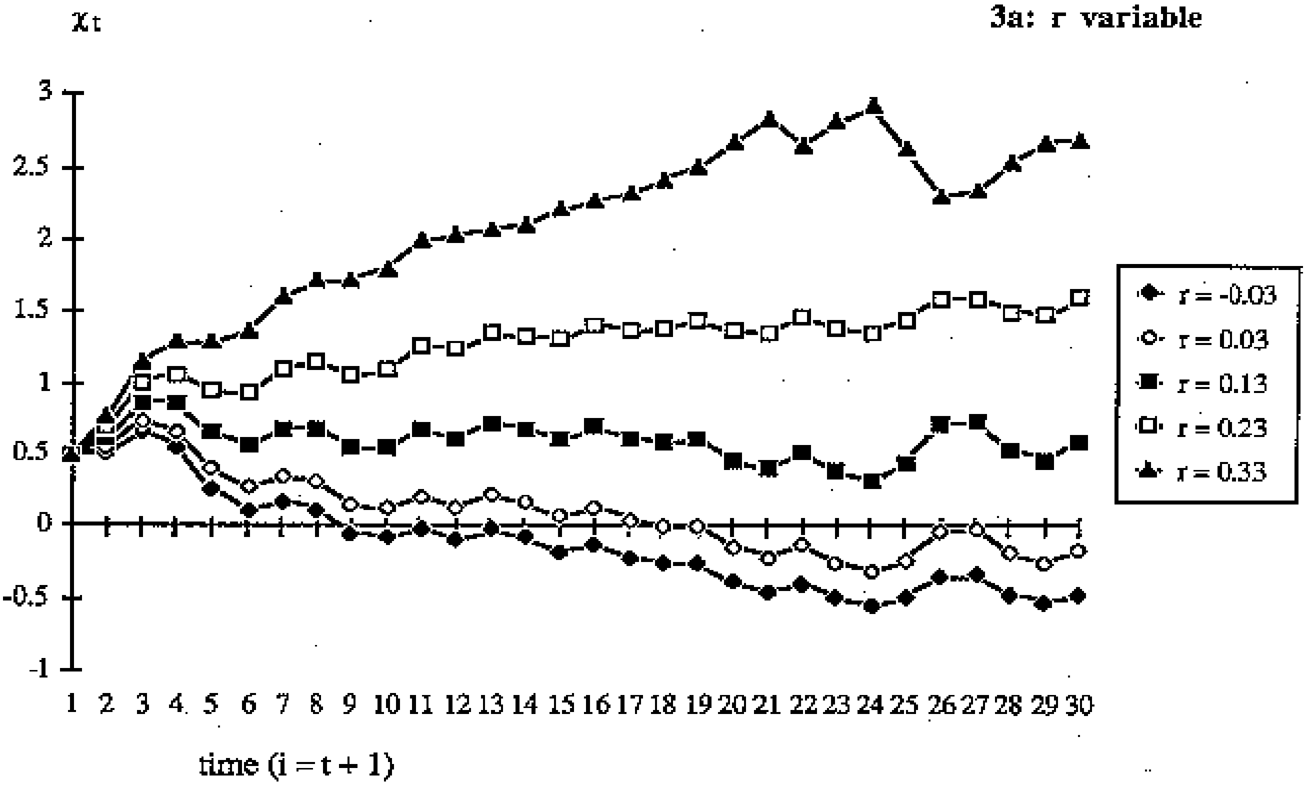
It is the economic parameters which determine whether the optimal level of stress on the ecological system is or is not sustainable. Since the economic parameters determine the optimal grazing pressure relative to the maximum sustainable grazing pressure, they also determine whether the stress imposed on the ecological system is sustainable in the absence of climatic shocks, and if it is sustainable in this sense, what magnitude of shocks can be absorbed without degrading the resource to the point where the herd collapses. Put another way, whether the optimal path for the state variables is stable in the face of climatic perturbation, depends on the value of the economic parameters relative to what may be termed the ecologically sustainable range. Given the dynamics of the ecological system, there exists a range for each non-ecological parameter over which the state variables are at least locally stable. This implies that change in relative prices or the rate of discount can either inhibit or enhance the resilience of the bioeconomic system.

To illustrate these properties of the system, consider once again the simulation of section 4. The control policy maximizes welfare (farm profits) over an infinite time horizon, given a set of ecological and economic parameters. It has already been remarked that at the initial parameter values the economically optimal level of grazing pressure is stochastically sustainable, in the sense that it will not cause the collapse of the carrying capacity of the range. The application of the optimal control policy will not lead to positive ecological overgrazing. However, this is no longer true if the economic parameters are varied significantly from the initial values. Figures 3a, 3b, 3c and 3d describe the ecological overgrazing indicated by a range of economic parameter values, allowing both for positive and negative values. The inclusion of negative values covers the case where zero-priced pastoral inputs are subsidized - which is not as uncommon as might be supposed.

Discussion of the implications of this exercise is deferred to the next section. At this stage the results of the exercise are merely reported. Figure 3a shows the effects of varying the range rental or range user fee from the initial value of 0.03. The degree of ecological overgrazing is shown to vary directly with the range user fee: a range user subsidy resulting in the 'understocking' of the range, and an an increase in range user fees raises the optimal level of grazing pressure. Indeed, in this example it turns out that a small rise in range user fees leads to a progressive fall in both the carrying capacity of the range and herd size under an optimal policy. Figure 3b shows, by contrast, that the effect of an increase in the maintenance cost of livestock is exactly the opposite: the degree of ecological overgrazing varying inversely with maintenance cost. Moreover, the lower the cost, the greater the relative variance in the degree of ecological overgrazing. In this case, the subsidizing of herd maintenance costs is associated with positive but highly fluctuating levels of ecological overgrazing. At the other extreme, very sharp increases in herd maintenance cost are associated with the progressive extinction of the herd through unsustainable levels of offtake. Figure 3c indicates an a very similar relation between the discount (interest) rate and the optimal level of grazing pressure. Higher rates of discount encourage higher levels of offtake now, leaving herd densities lower in the future. Increasing rates of discount reduce the pressure on the range. Finally, figure 3d indicates that the level of ecological overgrazing is not sensitive to change in the ouput price. Certainly, ecological overgrazing tends to increase as output prices rise, but in this example the increase is not significant over the range of prices tested.

Figure 3: Time paths for $\{\chi_t\}$

3a: r variable



3b: c variable

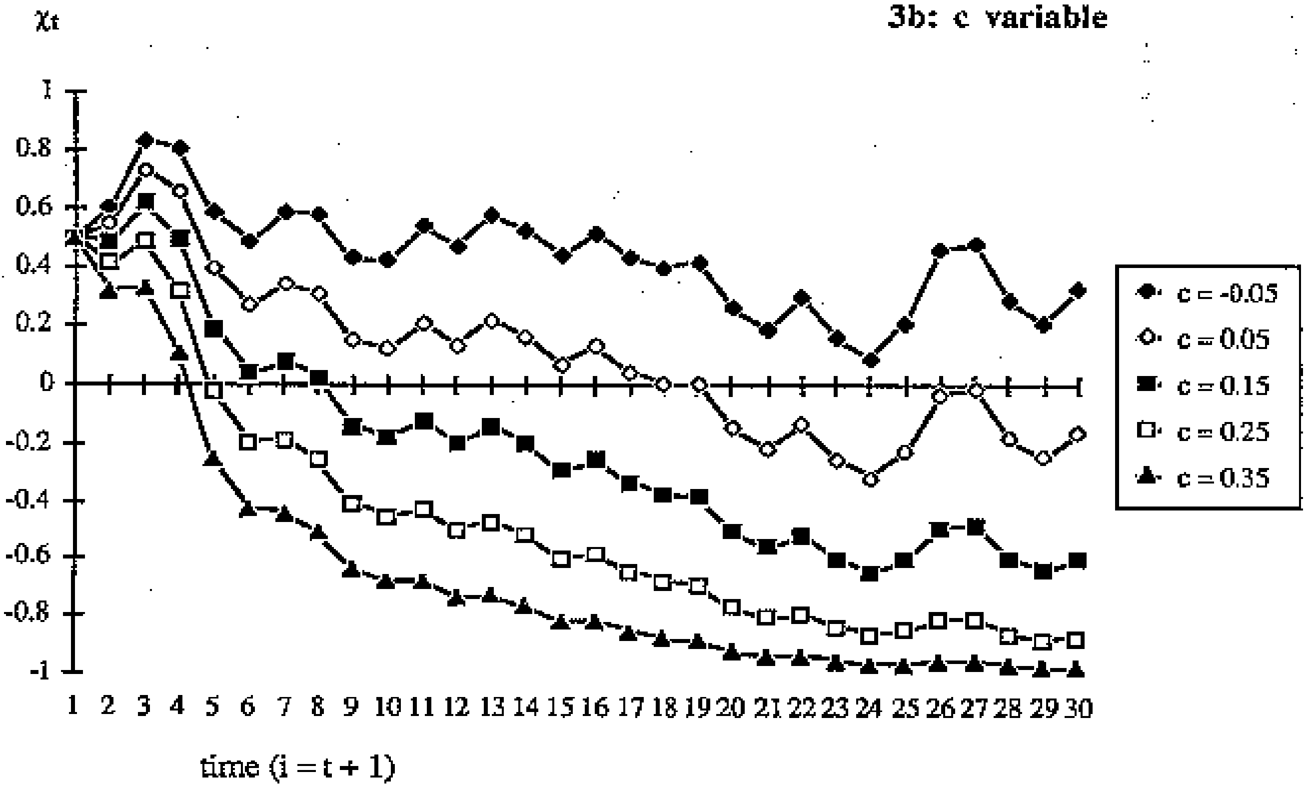
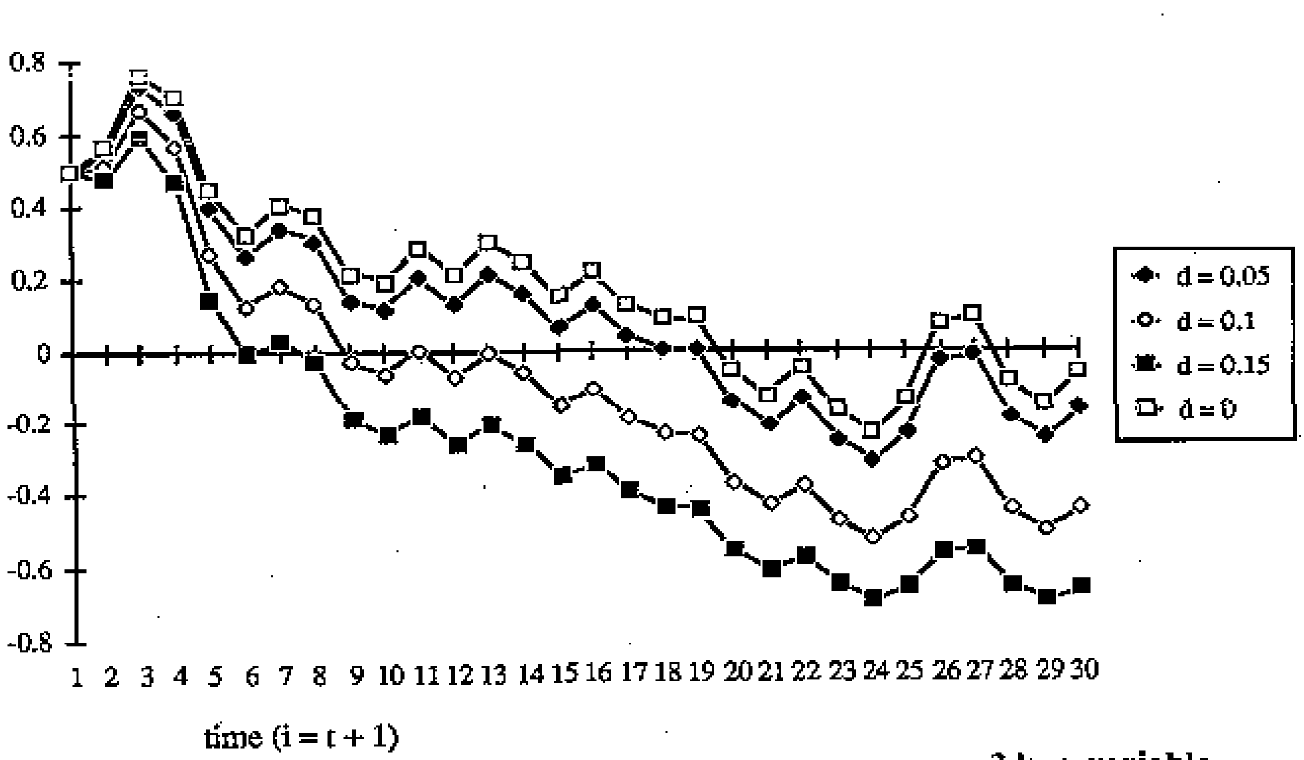
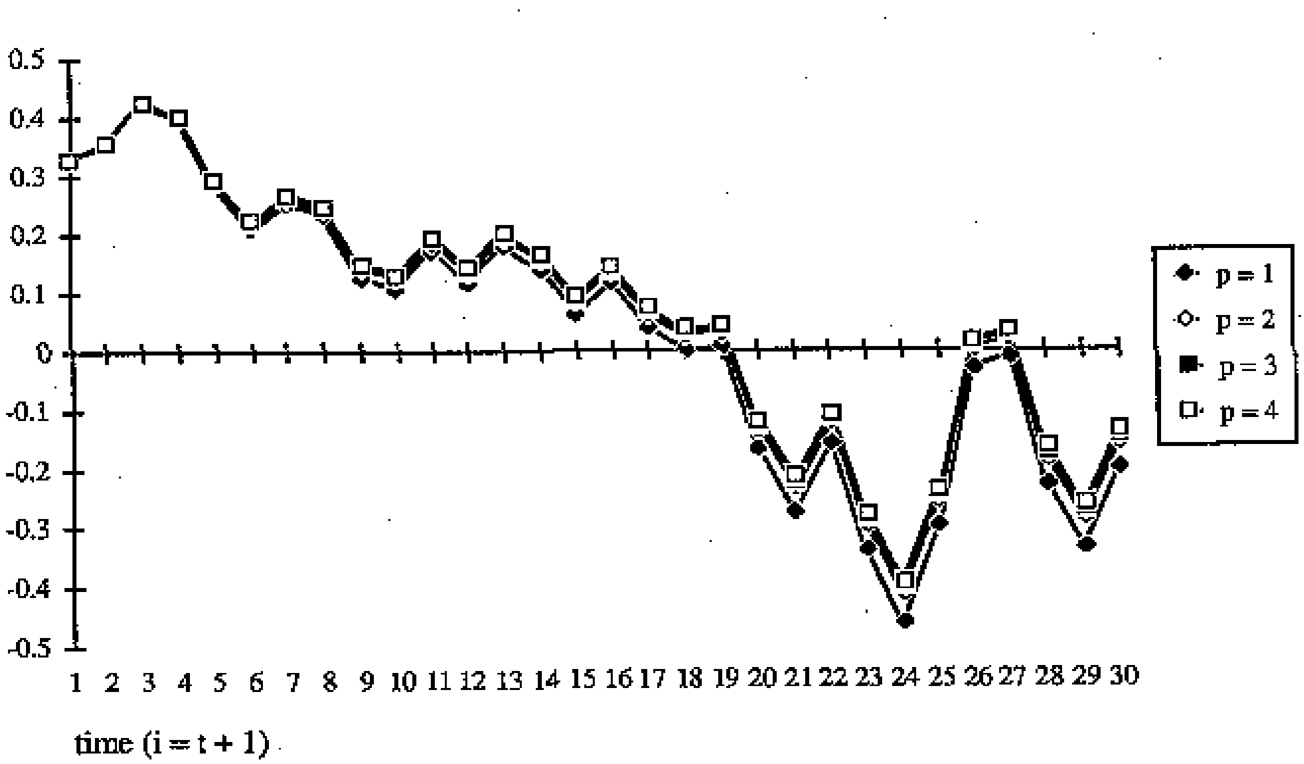


Figure 3: Time paths for {χt}

3c: δ variable



3d: p variable

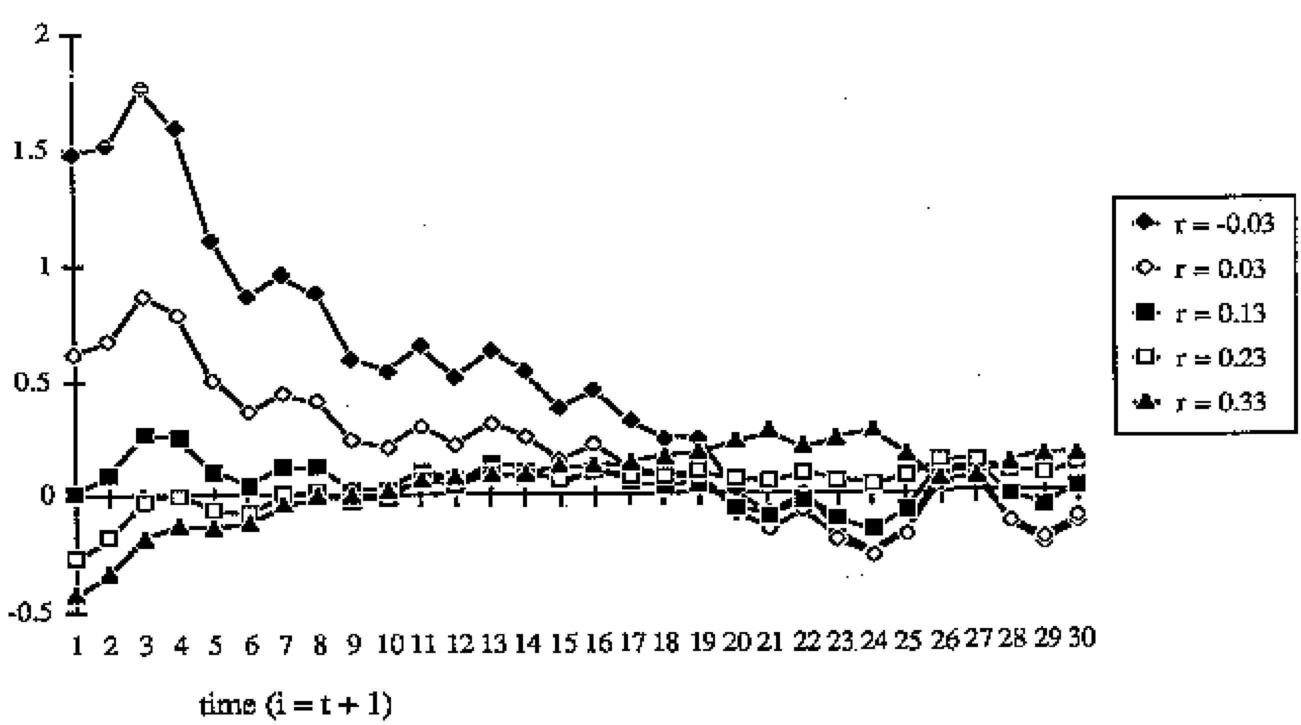


The second measure of overgrazing of interest in this paper is the level of economic overgrazing, ξ_t, which depends on the ratio of actual to optimal grazing pressure. The sequence $\{\xi_t\}$ describes the development of actual herd densities relative to the optimal density, and so sheds light on a rather different property of the bioeconomic system. Recall that $\{\chi_t\}$ describes the evolution of $\{\psi_t\}$ with respect to a fixed magnitude - the maximum sustainable grazing pressure. But the nature of the economic problem is such that the optimal level of grazing pressure is itself a function of the current state of the range and the herd. Consequently, $\{\psi_t^*\}$ evolves with the system, and may rise or fall as the state of the range or herd changes. The sequence $\{\xi_t\}$ accordingly indicates how the control process adapts to the evolution of optimal grazing pressure associated with the deterioration or enhancement of the carrying capacity of the range. But it also shows how differences in the optimal grazing pressure associated with differences in the economic environment impose greater or lesser stress on the natural environment, and so on the ability of the ecological system to accommodate climatic shocks.

Once again, these properties of the system are illustrated by extending the exercise reported in section 4 - in this case to consider the effect on $\{\xi_t\}$ of a range of economic parameter values. The results are shown in figure 4a, 4b, and 4c below. Figures 4a and 4b report the effect of changes in the value of range user fees and herd maintenance costs on the convergence of $\{\xi_t\}$, and illustrate the common feature of these experiments. Since different parameter values generate different optimal levels of grazing pressure, they will be associated with differences both in initial levels of economic overgrazing, and in the adjustment paths required to bring the ratio between the state variables into line with their optimal values. Because lower values of r are associated with lower optimal levels of grazing pressure, for example, they are also associated with greater initial economic overgrazing, and with greater required adjustments in the state variables. In addition, the more extreme the initial level of economic overgrazing (or understocking), the greater the change in the target values of the state variables along the way and the less stable is the convergence path for $\{\xi_t\}$. Figure 4c indicates the time paths for $\{\xi_t\}$ associated with a range of values for δ including one negative value. The positive values of δ reported here include only those for which there exist positive optimal levels of grazing pressure. Note that as δ rises, the optimal level of grazing pressure falls eventually becoming negative (passing through zero). Hence the initial level of economic overgrazing rises, eventually becoming negative (passing through infinity). For rates of discount higher than the maximum potential growth rate of the bioeconomic system, the herd is optimally extinguished over time. For the negative rate of discount reported here, the size of the

Figure 4: Time paths for $\{\xi_t\}$

4a: r variable



4b: c variable

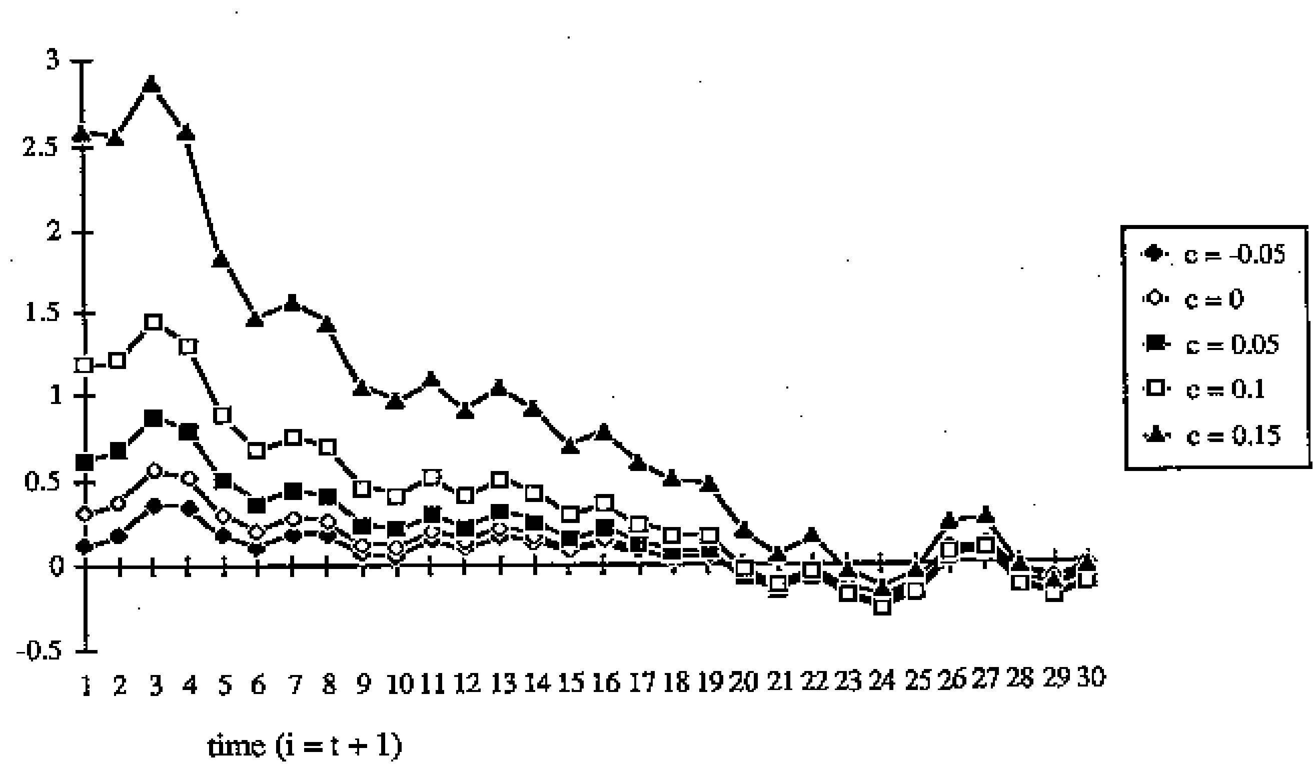
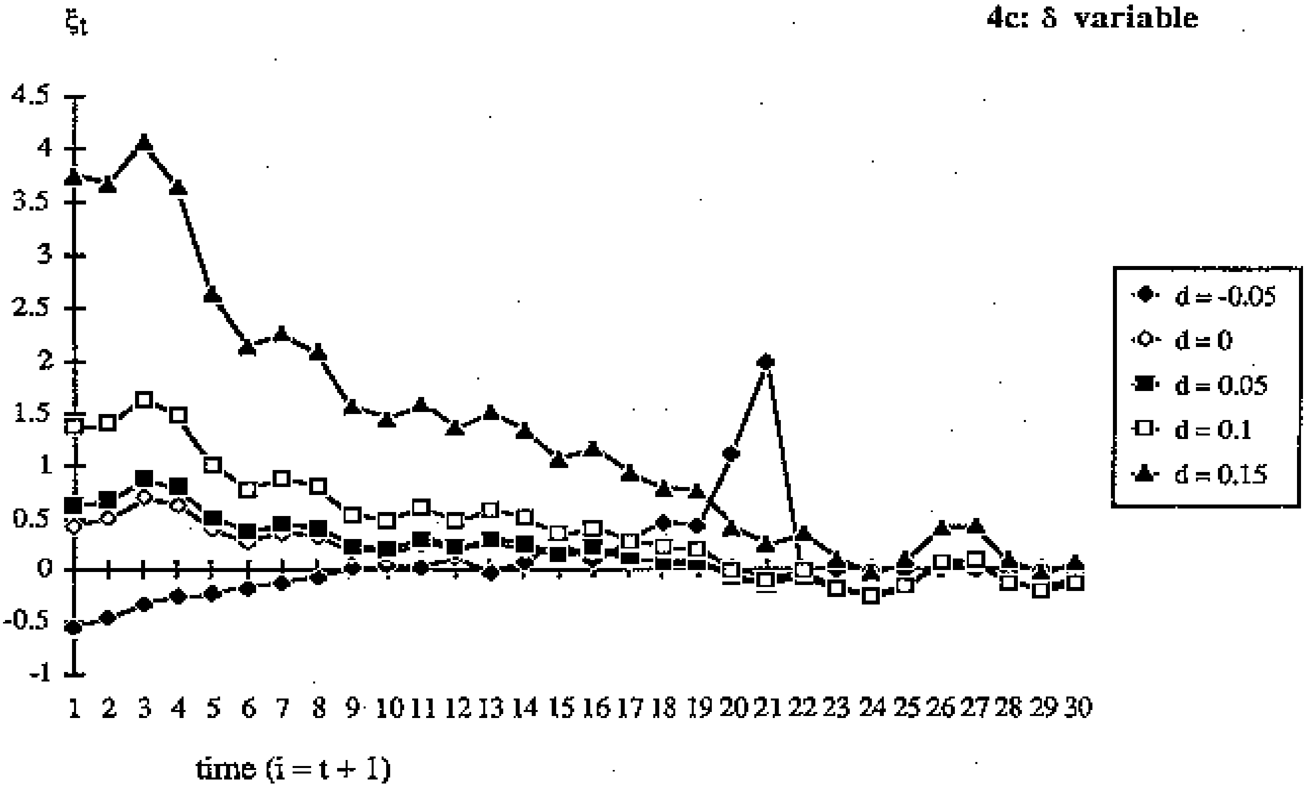


Figure 4: Time paths for $\{\xi_t\}$



herd relative to the current carrying capacity of the range is optimally increased to the point where the range collapses and the herd is similarly driven to extinction.

6 Stress, shock and the discount rate

To illustrate the implications of the stress levels imposed by different economic environments for the ability of the system to withstand shocks, it is convenient to focus on this last case - the discount rate. There is already considerable debate about the role of the discount rate in assuring the sustainability of resource use, and some concern about the ambiguous effects of a change in that rate. It is beyond the scope of this paper to review these concerns, but they hinge on the balance between the myopia implicit in high rates of discount, and the demands on the stock of natural capital implicit in low discount rates. Pearce Markandya and Barbier [1989], for example, are critical of myopia-based arguments against high rates of discount on these grounds. They argue that low discount rates may not lead to more sustainable use of renewable resources since they will tend to promote higher levels of investment (higher levels of economic activity) which may increase the stress on the environment. The model developed here supports the view that discount rates that are 'too low' relative to the productivity of the system may be incompatible with the sustainable use of resources, given that the rate of discount and the optimal grazing pressure are inversely related. But discount rates that are 'too high' relative to the productivity of the system will have the same effect. A progressive rise in the discount rate will ultimately lead to the collapse of the herd: a progressive fall will ultimately lead to the collapse of the range. Since the productivity of the system is a function of both the economic and the ecological environments, the stress imposed on the ecological system by a given discount rate will depend not both on the mean values of the stochastic ecological parameters, and on the values of the remaining economic parameters. However, whether or not a given level of stress is sustainable will depend on the variance of those parameters. In this example, the current values of the ecological parameters are assumed to be normally distributed about the mean, and the higher the variance of that distribution, the greater the probability of extreme events or 'climatic shocks'. The higher the variance of the stochastic ecological parameters, therefore, the less 'sustainable' is any given value of the economic parameters.

Consider the effects of a change in the variance of herd growth rates on the tolerance of the system to different rates of discount. The first point to make is that higher variance in the ecological parameters implies higher variance in the optimal time path for the state variables under any economic environment. Table 3 reports values for $\{\psi_t\}$ under different assumptions about the variance of α_t given the values of the economic parameters assumed in section 4: that is, for $p = 1$, $c = 0.05$, $r = 0.03$, $\delta = 0.05$, $\underline{\alpha} = 0.24$, $\underline{\beta} = 0.03$ and $\underline{\gamma} = 0.03$. At these parameter values the fluctuations in grazing pressure associated with the higher variance in α_t are sustainable in the sense that neither of the state variables collapses. However, consider the effects of a change in the discount rate in an ecological environment characterized by the higher dispersion in the current values of α_t reported in table 3.

Table 3: Time paths for $\{\psi_t\}$: $\sigma_\alpha{}^2$ variable

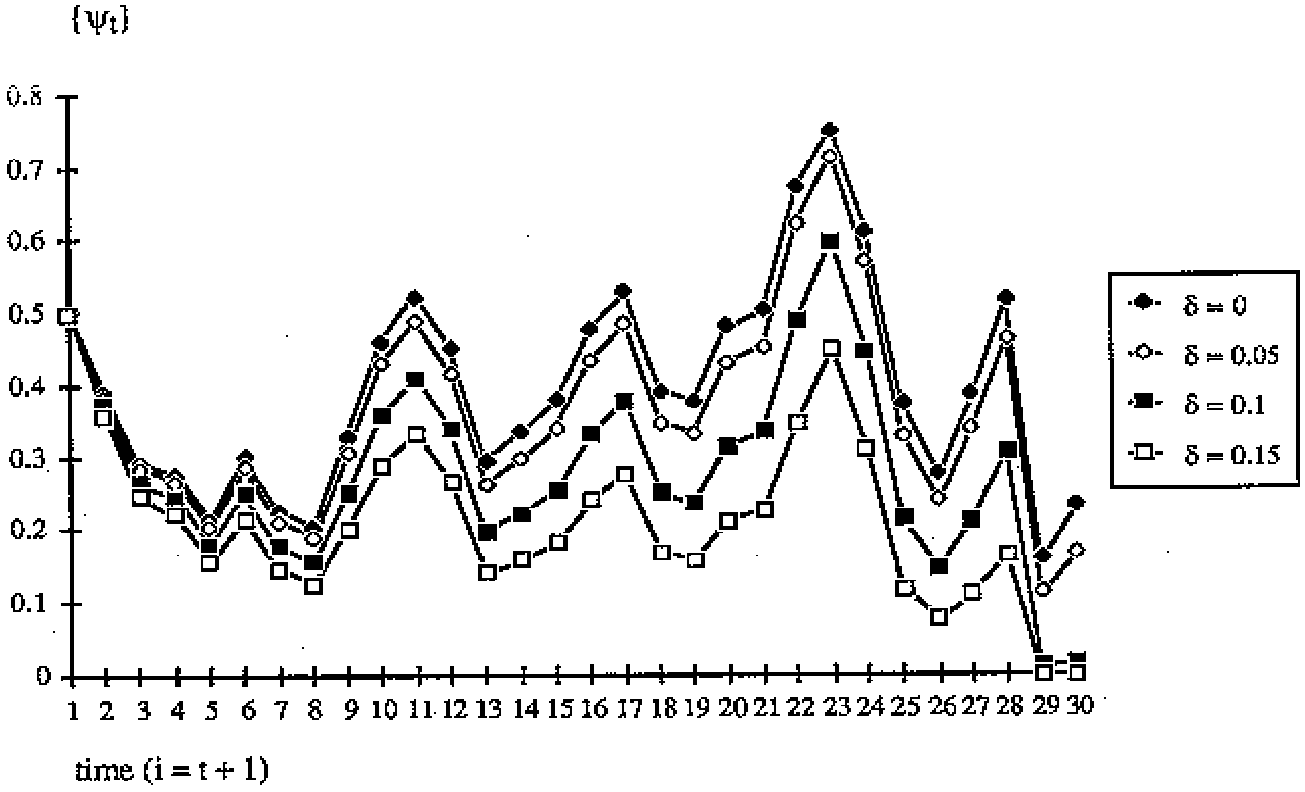
time	$\sigma_\alpha{}^2 = 0$	$\sigma_\alpha{}^2 = 0.0225$	$\sigma_\alpha{}^2 = 0.09$	$\sigma_\alpha{}^2 = 0.2025$
1	0.500	0.500	0.500	0.500
2	0.482	0.506	0.520	0.383
3	0.463	0.432	0.467	0.284
4	0.448	0.458	0.630	0.268
5	0.436	0.569	0.646	0.203
6	0.425	0.460	0.497	0.287
7	0.419	0.485	0.411	0.211
8	0.408	0.505	0.435	0.189
9	0.398	0.445	0.475	0.305
10	0.387	0.419	0.420	0.429

Table 3: **Time paths for $\{\psi_t\}$: σ_α^2 variable** (continued)

time	$\sigma_\alpha^2 = 0$	$\sigma_\alpha^2 = 0.0225$	$\sigma_\alpha^2 = 0.09$	$\sigma_\alpha^2 = 0.2025$
11	0.387	0.416	0.429	0.489
12	0.380	0.372	0.348	0.416
13	0.374	0.358	0.390	0.264
14	0.367	0.299	0.432	0.298
15	0.365	0.332	0.419	0.340
16	0.362	0.341	0.361	0.433
17	0.358	0.302	0.229	0.484
18	0.355	0.311	0.250	0.348
19	0.354	0.381	0.349	0.331
20	0.352	0.321	0.265	0.431
21	0.351	0.334	0.222	0.454
22	0.347	0.298	0.156	0.622
23	0.344	0.323	0.170	0.711
24	0.341	0.359	0.144	0.568
25	0.337	0.375	0.162	0.330
26	0.333	0.350	0.126	0.239
27	0.331	0.354	0.142	0.338
28	0.329	0.326	0.191	0.464
29	0.327	0.313	0.198	0.111
30	0.328	0.216	0.238	0.167

Figure 5 graphs the time paths for $\{\psi_t\}$ under the same (non-negative) range of discount rates described in figures 3c and 4c, but subject to a variance of $\sigma_\alpha^2 = 0.2025$ in the growth rate of the herd. As these two figures make clear, while higher discount rates are associated with optimal levels of grazing pressure well below the maximum sustainable level, the herd is not threatened with extinction under the original dispersion of $\{\alpha_t\}$. The same is not true under a sequence $\{\alpha_t\}$ with the higher variance. In this case an optimal policy based on the expected rather than the actual value of α_t cannot contain fluctuations in the size of the herd. However, whether those fluctuations threaten the extinction of the herd depends on the optimal grazing pressure. At a discount rate of 10 per cent the optimal grazing pressure is such that the herd is driven close to extinction within the time horizon of interest. At a discount rate of 15 per cent the optimal grazing pressure is such that the herd is driven to extinction. Whereas the stress placed on the ecological system by an economic environment including a 15 per cent discount rate is sustainable given the level of shocks expected under the original dispersion of α_t, it is unsustainable under the wider dispersion tested here. This suggests that considerable caution needs to be exercised in judging whether a given rate of discount (or a given value for any other economic parameter) is consistent with the sustainable use of environmental resources. It is the net effect of the economic environment that matters.

Figure 5: Time paths for $\{\psi_t\}$ at $\sigma_\alpha^2 = 0.2025$: δ variable



7 Concluding remarks

In recent years, considerable attention has been paid to the adverse environmental effects of the agricultural price regime in the low income countries. There is a very strong argument in the literature that the level of agricultural output prices is one reason for the degradation of arable and pastoral land in many low income countries, and that this is particularly true of sub-Saharan Africa. It is claimed that the agricultural sector in this region has been systematically disadvantaged by high levels of taxation of agricultural exports, financial repression, artificially depressed procurement prices and the protection of the industrial sector, and that this has prompted resource users to overexploit their natural environment. It is also claimed that liberalization of the agricultural price regime will contribute to the reversal of this trend. These claims raise a general problem that has not received much explicit attention. In what sense may an economic environment be incompatible with the sustainable use of natural resources drawn from sensitive ecological systems? Clearly, there are a number of relevant results in the existing theory of exhaustible and renewable resources. The 'iron law of the discount rate', for example, reflects a powerful result on the relationship between the rate of discount and the optimal rate of depletion of a resource. By implication, this provides a theory of the upper sustainable limit of an important economic parameter. But there is no analogous implicit theory of the lower sustainable limit of the same parameter. Yet it is the suspicion that there exists a lower limit which has excited much recent concern in the literature. Nor is there any clear understanding of the way in which economic and ecological parameters interact to determine both the the sustainable limits on parameter values, and the nature of the adjustment path given changes in parameter values.

This paper considers how the implications of a given economic environment might be evaluated in the context of a bioeconomic model of resource use in a sensitive natural environment. At the most general level, the paper confirms that positive stochastic equilibrium values for the state and control variables may be identified only if the economic parameters of the system lie within ranges defined by the remaining economic and ecological parameter values. Parameter values outside of such ranges imply the collapse of the system. Moreover, the interdependence of the ecological and economic parameters means that change in one, changes the response of the system to the others. Put another way, at the most general level the paper confirms that the economic environment sets the limits on the sustainable use of resources in any natural environment.

In the context of the model discussed here, it is possible to be more precise about the role of particular economic parameters. What is especially interesting, is that a number of parameters turn out to have unexpected effects. In so far as the improvement of procurement prices raises agricultural incomes, the argument that liberalization of the price system generates the means to conserve the resource base is well founded. However, the argument that higher slaughter prices in the pastoral economy will, ceteris paribus, encourage higher rates of offtake, and so lower levels of grazing pressure is more open to question. Higher slaughter prices have very little impact either on grazing pressure or on the degree of ecological overgrazing, but such impact as they do have is opposite to that expected in the literature. The higher the slaughter price, the greater the grazing pressure. The reason for this is that higher slaughter prices reduce both the real product cost of herd maintenance and the real product cost of carrying capacity, and these have opposite effects on grazing pressure. The implication of this is that unless the cost of carrying capacity (a grazing fee) is zero, higher slaughter prices will not lead to a reversal of the degradation of rangeland.

The different effects of a change in the cost of carrying capacity, r, and herd maintenance, c, may seem counterintuitive and so require some explanation. A fall in r implies that the marginal rate paid for range of a given carrying capacity falls relative to the cost of livestock maintenance on that range, inducing substitution in favour of range of higher carrying capacity. A fall in c, on the other hand, implies a fall in the marginal cost of livestock maintenance relative to the cost of carrying capacity, inducing substitution in favour of livestock. Hence a rise in grazing fees and a fall in livestock maintenance costs will each have the effect of increasing grazing pressure. This finding may be of particular interest in sub-Saharan Africa where r is at or close to zero in most cases, since it suggests that grazing fees may not be the way to secure a reduction in grazing pressure - unless they are inversely related to the carrying capacity of the range.

The impact of a change in discount rates is less unexpected, higher discount rates encouraging higher offtake, so leading to lower grazing pressure. As has already been remarked this implies that excessive values of δ may lead to the extinction of the herd, while insufficient values of δ may lead to the collapse of the range through overstocking. As with the other economic parameters, the optimal solution to the economic problem will only be sustainable if δ falls within the admissible range. The point here is that the stochastic equilibrium of the pastoral economy is highly sensitive to its economic environment. It is assumed in this paper that the ecological system is globally stable, implying that the carrying capacity of the range will always recover in the long run at a rate defined by β. But the economic system is not globally stable. Perturbation of any of the system parameters beyond the admissible range will cause the system to crash. Moreover, the admissible range will be greater or lesser depending on the variance of the stochastic parameters: the higher the variance in the stochastic parameters, the narrower the admissible range in the economic parameters.

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David W Pearce, Anil Markandya and Edward B Barbier
Blueprint for a Green Economy, Earthscan, September 1989, £6.95 (third printing)

This book by the London Environmental Economics Centre was prepared as a report for the Department of Environment, as a follow up to the UK government's response to the Brundtland Report. Here it stated that: '...the UK fully intends to continue building on this approach (environmental improvement) and further to develop policies consistent with the concept of sustainable development.'
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